# Local convective heat transfer from a heated surface to an impinging, planar jet of water

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Abstract Surface temperature and heat flux distributions have been measured on a flat, upward facing, constant heat flux surface cooled by a planar, impinging water jet. Data are presented for jet velocities between 1.8 and 4.5 m s<sup>-1</sup>, fluid temperatures of 30, 40 and 50°C and heat fluxes between 0.25 and 1.00 MW m<sup>-2</sup>. Appropriate length and velocity scales are identified, and results for the stagnation and boundary layer flows are correlated by expressions of the form  $Nu = C Re^{n} Pr^{m}$ . Measured stagnation convection coefficients exceed those predicted by a laminar flow analysis, and differences are attributed to the existence of free stream turbulence. Data are sensitive to variations in the stagnation flow heat transfer when there is free stream turbulence.

## INTRODUCTION

PLANAR jets of water are widely used to cool steel strip from hot rolling to coiling temperatures, and liquid jets are well suited for dissipating heat generated by microelectronic circuits. However, although transport processes in impinging gas jets have been studied extensively, the data base and predictive methods for liquid jets are relatively undeveloped. Clearly, optimization of current applications and the development of future applications for liquid jet cooling will require an improved knowledge of transport processes in impinging liquid jets. Although dimensional analysis suggests that liquid jet cooling can be estimated with existing gas jet correlations, experimental verification is required. Moreover, gas jet correlations can be applied only to submerged liquid jets.

A submerged jet issues into a miscible fluid of similar density and viscosity, and vigorous momentum exchange results in expansion of the jet, a reduction in the jet velocity, and turbulation of the fluid. Examples include gas jets in a gaseous ambient and liquid jets in a liquid ambient. In contrast, propagation of a freesurface jet is virtually unimpeded by an immiscible ambient fluid of substantially lower density and viscosity, and jet momentum can be efficiently delivered to and redirected along a solid surface. A free-surface jet is also subject to acceleration by gravity, with a corresponding reduction or increase in the jet crosssectional area.

Stagnation heat transfer to an impinging jet has been modeled under the simplifying assumptions of a

laminar and uniform flow in the free stream. A planar jet impinging on a flat surface resembles the wedge or Falkner-Skan class of self similar laminar boundary layer flows. A similarity transformation permits solution of the momentum boundary layer equation for flow over a wedge in which the free stream velocity is parallel to the wedge axis [1]. Computed wedge flow velocity profiles [2] have been used to develop solutions to the boundary layer energy equation for several thermal boundary conditions [3, 4]. Hiemenz [5] derived a local solution to the Navier-Stokes equations, valid only in a small but finite region near the stagnation point of a two-dimensional laminar flow on an arbitrary two-dimensional body. This analysis may be applied without hesitation near the stagnation line of a finite width, planar jet impinging on a flat plate. In contrast, because this flow is not uniform, the Falkner-Skan analysis is not precisely applicable. However, when  $\beta = 1$  (an interior wedge angle of  $\pi$ ), the Hiemenz equation and boundary conditions are identical to those of the Falkner-Skan flow. Hence, the similarity analysis for laminar flow impinging on an open wedge does apply near the stagnation line, even though the analysis does not apply downstream. Existing solutions to the wedge flow problem may therefore be used to obtain the stagnation line heat transfer coefficient.

Levy [3] has shown that heat transfer coefficients for Falkner–Skan flow over constant temperature and heat flux surfaces are the same when  $\beta = 1$ . This result permits use of Evan's analysis [4] for a constant surface temperature to predict the stagnation flow heat transfer on a uniform heat flux surface. Evans reports constant values of  $Nu_x/Re_x^{1/2}$  for Prandtl numbers between 0.7 and 10. These values are closely approximated by

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NOMENCLATURI
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С	velocity	gradient,	$du_{\delta}/dx$
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- $c_p$  specific heat
- *D* diameter of a circular heated area
- d diameter of a circular nozzle
- $d_{w}$  wire diameter G volume flow rate
- *H* nozzle discharge to heater spacing
- *h* heat transfer coefficient
- $k_{\rm f}$  thermal conductivity of fluid
- $Nu_d$  Nusselt number,  $hd/k_f$
- $Nu_{\rm j}$  Nusselt number,  $hw_{\rm j}/k_{\rm f}$
- $Nu_0$   $Nu_j$  for a flow with zero free stream turbulence, equation (4)
- $Nu_r$  Nusselt number,  $hr/k_f$
- $Nu_x$  Nusselt number,  $hx/k_f$
- *Pr* Prandtl number,  $\mu c_p/k_f$
- q" heat flux
- $Re_d$  nozzle Reynolds number,  $V_j d/v$
- $Re_i$  jet Reynolds number,  $V_i w_i / v$
- $Re_n$  nozzle Reynolds number,  $V_n w_n/v$
- $Re_r$  plate Reynolds number based on jet velocity,  $V_i r/v$
- $Re_x$  plate Reynolds number based on jet velocity,  $V_y x/v$
- $Re_{x^*}$  plate Reynolds number based on local velocity,  $u_{\delta}x/v$
- $Re_{x,c}$ ,  $Re_{x^*,c}$  critical Reynolds numbers
- r radial coordinate measured from the stagnation point of a circular jet; exponent controlling heat flux distribution
- St Stanton number, Nu/(Re Pr)
- $T_{\rm f}$  fluid temperature

- $T_{\rm w}$  surface temperature
- *Tu* turbulence intensity
- *u x*-component of velocity
- $u_{\delta}$  local free stream value of u,  $u(x, \delta(x))$
- $V_{\rm j}$  velocity of the impinging jet
- $V_{\rm n}$  jet velocity at the nozzle discharge
- $w_j$  width of the impinging jet
- $w_n$  jet width at the nozzle discharge
- *x* streamwise position measured from the stagnation plane
- *x*<sub>c</sub> value of *x* for incipient boundary layer turbulence
- y coordinate normal to the heated surface.

## Greek symbols

- β measure of interior wedge angle for a Falkner–Skan flow; velocity distribution parameter, 2m/(m+1)where  $m = (x/u_{\delta})(du_{\delta}/dx)$
- γ exponent controlling temperature distribution
- $\delta$  velocity boundary layer thickness; uncertainty interval
- $\delta_t$  thermal boundary layer thickness
- $\mu$  dynamic viscosity
- v kinematic viscosity
- $\omega_i$  vorticity associated with free stream turbulence
- $\omega_{np}$  vorticity not in the orientation preferred for amplification
- $\omega_{\rm p}$  vorticity in the orientation preferred for amplification.

$$\frac{Nu_x}{Re_{x^*}^{1/2}} = 0.569 Pr^{0.376}.$$
 (1)

The coefficient (0.569) and Prandtl number exponent (0.376) were evaluated by forcing the function to agree with the theoretical results of Evans [4]. The Falkner–Skan and Hiemenz analyses assume  $u_{\delta} = Cx$ , where C is the velocity gradient  $du_{\delta}/dx$ . When primitive variables are substituted for the Nusselt and Reynolds numbers, the heat transfer coefficient is shown to be independent of position (Table 1).

Local heat transfer downstream of the stagnation region has been predicted by boundary layer analyses [6–8]. The boundary layer equations are readily solved once the streamwise pressure variation is known. Pressure in the velocity boundary layer has been evaluated by solving Euler's equation for momentum transport in the inviscid free stream [6, 7]; the pressure field has also been determined experimentally [7, 8]. Schach [9, 10] studied the streamwise pressure distribution at the wall for impinging circular and planar jets. The flow field and pressure distribution were evaluated by solving Euler's equation for an impinging slug flow, and results agreed with computed values to within  $\pm 2\%$ .

The analysis of Inada et al. [7] is based on the assumption that the local heat transfer coefficient at a distance  $x/w_i$  from the stagnation line is equivalent to that for a wedge flow having the same velocity gradient  $(du_{\delta}/dx)$  at the same distance from the stagnation line. Levy [3] has derived an expression for the wedge flow Nusselt number for power function variations of free stream velocity (wedge flows) and temperature or heat flux. Inada et al. [7] used Strand's [11] analysis of velocity fields in a planar impinging jet to compute equivalent values of  $\beta$  for flow at the position  $x/w_i$ . They predicted local heat transfer in the laminar boundary layer by supplying a local value of  $\beta$  to Levy's result. The model shows that heat transfer is affected strongly as the nozzle is moved in close proximity to the plate. Values of  $H/w_n < 5$  caused

the maximum heat transfer coefficient to occur near  $x/w_n = 0.5$ , instead of at the stagnation line. Movement of the nozzle beyond  $H/w_n = 5$  had little effect on the local heat transfer coefficient. Miyazaki and Silberman [6] corroborated this result.

Metzger et al. [12] measured the stagnation heat transfer to impinging, circular water and oil jets, and data were correlated by a least squares fit. Hatta et al. [13] performed quenching experiments in which a circular water jet cools a steel plate from 900°C, and radial variations in the non-boiling convection coefficients were correlated. Measured impingement cooling data for planar jets have also been reported [7, 8, 14-16]. Ishigai et al. [14] plotted the results of their steady and quenching experiments but did not suggest a correlation. Miyasaka and Inada [15] reported the stagnation line convection coefficient as  $h = 1.03 Pr^{1/3} k_{\rm f} (C/v)^{1/2}$  for  $10^4 \le Re_{\rm i} \le 1.4 \times 10^5$ . The coefficient predicted by laminar boundary layer analysis is 0.569 (equation (1)). Later, however, stagnation line data generated by Inada et al. [7] with the same heat source and  $Re_i = 940$  was in agreement with the authors' aforementioned analytical model. Only McMurray et al. [16] and Zumbrunnen et al. [17] reported laminar convection coefficient data downstream of the stagnation line. Miyasaka and Inada [15] and McMurray et al. [16] measured heat transfer in the turbulent boundary layer. The correlations are summarized in Table 1.

Measured stagnation line convection coefficients are in poor agreement with the analytical solution for a laminar, impinging jet. Data reported by different investigators are also not in agreement. Experimentalists have shown that, when the pressure gradient in a boundary layer flow is zero, increased free stream turbulence has no influence on heat transfer other than reduction of the critical Reynolds number [18–20]. When a pressure gradient is present, however, free stream turbulence can enhance heat transfer in the laminar boundary layer by amounts ranging from a few percent to more than double [20-22]. The phenomenon is most commonly observed in stagnation flows, since the pressure gradient can be large in the stagnation region. Increasing the free stream turbulence intensity from 0 to 3% can cause an 80% increase in the stagnation heat transfer from a cylinder in cross flow [21, 23–26]. Similar results are reported for the stagnation region of impinging jets [27-29]. Experimental data consistently show that an increase in the free stream turbulence intensity increases the enhancement factor, which is a multiplier for the laminar solution that will yield the measured result.

Kestin [21] makes a dimensional argument that, in addition to the Reynolds number, turbulence intensity and scale must be matched in order to achieve dynamic similitude in a turbulent flow. When the turbulence scale is small with respect to an appropriate length scale, such as boundary layer thickness, he

Table 1. Summary of studies addressing heat transfer to an impinging free-surface jet

Authors	Problem studied	Result	Comments
Levy [3] Miyasaka	P, W, A P, S, E, H	Laminar boundary layer $h = 0.569k_f (C/v)^{1/2} Pr^{0.376}$ $h = 1.03k_f (C/v)^{1/2} Pr^{1/3}$	• similarity solution • $w_j = 1$ cm
Levy [3] and Inada <i>et al.</i> [7]	P, L, A	$Nu_{x} = C(\gamma, \beta) Re_{x^{\star}}^{1/2} Pr^{m(\beta)}$	<ul> <li>C(γ, β) and m(β) are unique to H/w<sub>j</sub></li> <li>valid for T<sub>w</sub> = Ax<sup>7</sup> or q = Dx<sup>7</sup> where r = γ - [(1-β)/(2-β)]</li> <li>when x/w<sub>j</sub> ≥ 5 and r = 0, 1 ≥ β ≥ 0 [C(1) = 0.57] ≥ C(β) ≥ [C(0) = 0.46] [m(1) = 0.38] ≥ m(β) ≥ [m(0) = 0.34]</li> </ul>
Miyazaki and Silberman [6]	P, L, A, T	$Nu_{\rm j}=f(x/w_{\rm j})Re_{\rm j}^{1/2}$	• boundary layer analysis • $f(x w)$ is unique to $H w$ and $Pr$
McMurray <i>et al.</i>	P, L, E, H	$Nu_x = 0.73 Re_{x^*}^{1/2} Pr^{1/3}$	• $w_i = 0.64$ cm • $w_i = 0.64$ cm
Zumbrunnen <i>et al.</i> [8, 17]	P, L, E, Q	$Nu_{j} = f(x/w_{j})Re_{j}^{n}Pr^{0.4}$	• when $w_j = 1$ cm, $n = 0.608$ and $[f(0) = 0.33] \ge f(x/w_j) \ge [f(10) = 0.109]$ • when $w_j = 2.03$ cm, $n = 0.666$ and $[f(0) = 0.149] \ge [f(x/w_j) \ge [f(5) = 0.060]$
Metzger <i>et al.</i> [12]	C, Av, E, T	$Nu_{\rm d} = 2.74 R e_{\rm d}^{0.348} P r^{0.487} (d/D)^{0.774}$	• $3 < d/D < 25$ • partial boundary layer turbulence is likely for large $d/D$
Hatta et al	CLEO	Turbulent boundary layer $N_{H} = 0.063 Re^{0.8} Pr^{1/3}$	$\bullet d = 1 \text{ cm}$
[13]	C, D, D, Q	114, 0.005AC, 11	
Miyasaka and Inada [15]	P, L, E, H	$Nu_{x} = 0.033 Re_{x^{*}}^{0.8} Pr^{4/3}$	• based only on data for $x/w_j = 2.5$ • unheated upstream flow
McMurray <i>et al.</i> [16]	P. L. E. H	$Nu_{x} = 0.037 Re_{x^*}^{0.8} Pr^{1/3}$	······································

P, planar jet; C, circular jet; W, wedge flow results; S, stagnation line result; L, local result; Av, average result; A, analytical study; E, experimental study; H, uniform heat flux; T, uniform temperature; Q, quench.

argues that the time averaged effect of turbulence scale on the local transport of mass, momentum, energy and species is negligible. For such cases Kestin concludes that the Nusselt number is a function of the Reynolds number, Prandtl number and free stream turbulence intensity, Nu = f(Re, Pr, Tu). This result has been widely affirmed by experimentalists, who have shown that functions of the form  $(Nu/Nu_0) =$ 1 + f(Re Tu) and  $(Nu/Re^{1/2}) = f(Tu Re^{1/2})$  are suitable for correlating data [25, 30, 31].

Interaction between the laminar boundary layer and oscillations in the free stream was proposed by early investigators as the mechanism for sensitizing stagnation flow heat transfer to free stream turbulence. Analyses of a laminar boundary layer with twodimensional, harmonic oscillations in the free stream did not predict, however, experimentally observed increases in heat transfer relative to values for steady, laminar stagnation flow [21, 32–34]. Sutera et al. [35] proposed that the mechanism for enhancement of stagnation flow heat transfer is amplification of vorticity caused by the stretching of vortex lines in the diverging flow. They developed a mathematical model for the case of steady, spatially periodic vorticity that is oriented parallel to the streamlines near the solid boundary. Only vorticity with this orientation will be amplified by stretching [5]. Sutera et al. predicted sensitivity of shear stress and heat transfer to the free stream vorticity. The effect on heat transfer was much greater than on shear stress. One calculation (Pr = 0.74) predicted only a 5% increase in shear stress when the heat transfer was increased by 26% over the case of no vorticity. This behavior is in agreement with results from the experimental portion of Sutera et al.'s study. Others have also confirmed Sutera et al.'s result [36-39].

Linear and non-linear stability analyses have been used to show that longitudinal vortices are generated when free stream vortex lines that are oriented in the same direction as the flow acceleration are amplified by stretching [40-42]. The existence of counter rotating longitudinal vortices lying close to but outside of the laminar stagnation boundary layer has been verified by flow visualization for laminar impinging jets [40, 43]. VanFossen and Simoneau [43] have shown that flow in the boundary layer between the longitudinal vortices and the wall is three-dimensional. Sheets of temperature sensitive liquid crystal were used to show variations in the surface temperature. Fluid was either directed toward or away from the wall between vortices, depending on the direction of rotation. The wall temperature was lowest, indicating a larger convection coefficient, where the vortices swept fluid toward the wall. However, since vorticity in a turbulent flow is unsteady in amplitude and orientation, the results of idealized studies involving steady vorticity cannot be applied directly to predict heat transfer to an impinging jet with free stream turbulence.

Experimentalists have established a phenomeno-

logical relationship between the convection coefficient in a laminar boundary layer with a pressure gradient and the free stream turbulence intensity. The pressure gradient is an important parameter because it is coupled to the velocity gradient in the free stream through Euler's equation. Free stream turbulence has no effect on heat transfer in a laminar boundary layer in the absence of a velocity gradient because the stretching and amplification of vorticity in the flow is caused by acceleration of the flow field along the length of a vortex line.

Sutera [36] has shown that the Prandtl number also affects the sensitivity of heat transfer in the laminar, stagnation boundary layer to free stream vorticity. The sensitivity was greater for Pr = 7 than for Pr = 0.7 or 100. In the limit of very large or very small Prandtl number, the effect of vorticity amplification on stagnation flow heat transfer is small. When  $Pr \rightarrow$ 0, the thermal boundary layer is much thicker than the hydrodynamic boundary layer ( $\delta_1 \gg \delta$ ), and since the temperature gradient near  $v = \delta$  is small, vortex induced mixing at this location will have little effect on heat transfer. As the Prandtl number increases and the thermal boundary layer thins, the sensitivity of heat transfer to perturbations of the velocity field caused by amplified free stream vorticity increases. Enhancement of impingement cooling does not, however, increase monotonically with Prandtl number. When  $Pr \rightarrow \infty$ , the thermal boundary layer is much thinner than the hydrodynamic boundary layer  $(\delta_1 \ll \delta)$ . Heat transfer will take place in a thin layer of fluid that is adjacent to the wall and well removed from vortex induced flow disturbances that lie near  $y = \delta$ .

This study addresses steady heat transfer from a uniform heat flux surface to a planar, free-surface water jet. The objectives are to (1) extend the data base, which was previously limited to McMurray *et al.*'s study, (2) reconcile differences in the length scales that have been chosen for various impingement cooling studies, and (3) examine the data for sensitivity to free stream turbulence parameters that theory and gas jet data suggest will affect heat transfer.

## EXPERIMENTAL METHODS

Heat transfer to an impinging water jet was computed from measured temperature, power and flow data. To facilitate these measurements, thin plate heaters were made from 304 stainless steel and Haynes alloy number 230, H (Fig. 1). Each plate was 260 mm long and 35.7 mm wide. The thickness was 0.381 mm for the 304 stainless steel and 0.635 mm for the Haynes 230. A 119 mm long portion of the plate was heated by an electric current, and when installed in the heater module, the plate was placed in tension to inhibit buckling (B, C). Air jets (A) prevent fluid from passing through the sliding contact between the heater (G) and the enclosure on which it rests (K).

A cartridge-like measurement module (Fig. 2) was



FIG. 1. Assembly drawing of the heater module.

designed for insertion into the cavity below the heater plate (Fig. 1). Temperatures were measured by spring loaded thermocouples at the dry face of the heater. The thermocouples were positioned at 5.08 mm intervals along a line that is parallel to the direction of fluid flow and is centered between the edges of the plate (Fig. 3). A metal filled silicone paste reduces thermal contact resistance between the thermocouple junction and the heater. The paste also electrically isolates the thermocouple circuits from the heater. Substrates A and B (Fig. 2), which were pressed against the heater by springs, insulated the dry face



FIG. 2. Assembly drawing of the apparatus for measuring the temperature distribution and voltage drop at the heater surface.

of the heater so that the corresponding surface heat flux was small compared to the heat flux at the wetted face. The temperature and heat flux at the wetted face were found by solving the steady heat equation for the plate.

Heat dissipation in the plate was controlled with a d.c. power supply, the peak output of which was 15 kW (1500 A and 10 V). The Joulean heat generation term in the energy equation for the plate was computed from the voltage drop across the heated section and the electrical resistivity of the heater material. The voltage drop across the heater was measured by a differential probe positioned along the heater midline.

Characterizing heat transfer to the impinging jet requires knowledge of the jet's temperature, velocity, width and thermophysical properties. The jet temperature, which was controlled by a heater and cooling coil in the water reservoir, was measured in a plenum directly above the nozzle entrance. The flow system could deliver between 1 and 7 1 s<sup>-1</sup> at temperatures between 20 and 60°C through a 10.2 mm by 105 mm nozzle discharge. A convergent nozzle, with a honeycomb straightener at the entrance, was used to minimize turbulence in the flow. Jet velocity and width were corrected for the effects of gravitational acceleration between the nozzle discharge and the impingement surface. Although the nozzle width was 10.2 mm for all experiments, the impinging jet width varied between 6.9 and 9.7 mm. The measurement system and data reduction procedures are described in detail elsewhere [44, 45].

The standard procedure for computing experimental uncertainty in single sample experiments [46] was applied to the conditions of this study, and uncertainties for the quantities of interest are as follows: convection coefficient,  $100(\delta h/h) \approx \pm 12\%$ ; Nusselt number,  $100(\delta Nu/Nu) \approx \pm 12\%$ ; jet velocity,  $100(\delta V_j/V_j) \approx \pm 3\%$ ; jet width,  $100(\delta w_j/w_j) \approx \pm 3\%$ ; Reynolds number,  $100(\delta Re_j/Re_j) \approx \pm 5\%$ , and  $100(\delta Re_{x*}/Re_{x*}) \approx \pm 5\%$ . The uncertainties are based on 20:1 odds and do not account for variations caused by system specific parameters, such as free stream turbulence intensity. Convection heat transfer data were repeatable to within  $\pm 5\%$ , well within the uncertainty band.

## **RESULTS AND DISCUSSION**

Impingement cooling data have been gathered over the full range of experimental conditions permitted by the apparatus. Convection heat transfer is influenced by controllable independent variables such as jet velocity, fluid temperature and heat flux, as well as by other parameters such as free stream turbulence. Results of this study focus on non-boiling convection data and are concerned with delineating important heat transfer mechanisms in terms of the controlled variables, q'',  $V_j$  and  $T_i$ , and the free stream turbulence, which varied but was not controlled.



FIG. 3. Schematic diagram of the experiment.

#### Temperature distribution

Surface temperature variations depicted in Figs. 4 and 5 are typical for non-boiling impingement cooling. The symbols represent reduced data points corresponding to the position  $x/w_j$  of temperature sensors placed on the insulated face of the heater. Prior to solving the heat equation for the heater plate, measured temperatures were smoothed by a least squares cubic spline fit.

Since the flow velocity is zero at the stagnation line, the boundary layer immediately downstream of that line will be laminar. Heat is added to the fluid as it moves away from the stagnation line, causing the thermal boundary layer thickness and the resistance to heat transfer between the surface and the free stream to increase. Hence, with a nearly uniform heat flux, the surface temperature must increase with increasing  $x/w_j$ , if the boundary layer remains laminar. Boundary layer turbulence is initiated by free stream flow disturbances, surface roughness, acoustic noise, trip wires or other sources, and transition consists of a sequence of events which are distributed in the streamwise direction. Boundary layer turbulence enhances fluid mixing near the wall, thereby increasing local heat transfer.

Surface temperatures between the onset and completion of the transition to a fully turbulent boundary layer are affected by a turbulence induced increase in the local convection coefficient. Growth of the thermal boundary layer as heat is added to the flow continues



FIG. 4. Effect of jet velocity on local surface temperature when q'' = 0.25 MW m<sup>-2</sup> and  $T_f = 30^{\circ}$ C.



FIG. 5. Effect of jet temperature on local surface temperature when  $q'' = 0.25 \text{ MW m}^{-2}$  and  $V_j = 2.5 \text{ m s}^{-1}$ .

to be a mechanism for increased impedance to heat transfer. Turbulent mixing, however, becomes increasingly vigorous in the streamwise direction, acting to reduce the impedance to heat transfer by increasing the rate at which fluid near the wall is exchanged with fluid in the free stream.

Incipience of the transition to turbulence is marked by a reduction in the streamwise rate of surface temperature increase. When turbulent mixing near the wall becomes sufficiently vigorous to reduce the resistance to heat transfer across the thermal boundary layer, the surface temperature declines from a local maximum. The rate of decline depends on how rapidly turbulent mixing increases, and the surface temperature continues to decrease until the flow is fully turbulent. As streamwise increases in turbulent mixing near the wall diminish, the resistance to heat transfer between the wall and the free stream, and hence the surface temperature, increases.

Although a local maximum and minimum in temperature do not correspond exactly to incipience and full development of a turbulent boundary layer, they are commonly used to estimate these events. A unique (critical) value of the Reynolds number,  $Re_{x^*c} =$  $u_{\delta}x_{\rm c}/v$ , should indicate the onset of boundary layer turbulence. As the jet velocity increases,  $Re_{x^*,c}$  can only be constant if the position for transition,  $x_c/$  $w_i$ , moves toward the stagnation line. A similar argument applies to the case of increasing temperature in the boundary layer, either by increasing the free stream temperature or the heat flux. Because the viscosity of water decreases as temperature increases,  $x_{\rm c}/w_{\rm i}$  must decrease as the free stream temperature or the heat flux are increased. If  $x_c/w_i$  is approximated as the position of the maximum temperature occurring downstream of the stagnation line, the expected response of  $Re_{x^*,c}$  to changes in  $V_i$  and  $T_f$  is clearly seen in the results of Figs. 4 and 5.

The average critical Reynolds number for 14 nonboiling heat transfer experiments is  $Re_{x^*,c} = 3.6 \times 10^5$ , and the standard deviation is  $\pm 2.7 \times 10^4$  ( $\pm 7.5\%$ ). McMurray *et al.* [16], who previously measured steady heat transfer to a planar, impinging jet, also observed a critical Reynolds number of  $Re_{x^*,c} = 3.6$  $\times 10^5$ . Zumbrunnen [8, 17], however, reported  $Re_{x,c} = 1.9 \times 10^5$ , where  $Re_{x^*,c} \approx Re_{x,c}$  if  $x_c/w_j > 3$ . Zumbrunnen performed quenching (transient) experiments in which temperature sensors were mounted flush with the surface of a metal plate.

For the conditions of this study, the heater was too short for complete development of the turbulent boundary layer. Moreover, for  $T_f = 30^{\circ}$ C and  $V_j \le 2.1 \text{ m s}^{-1}$ , the Reynolds number at the trailing edge of the heater did not reach the critical value.

#### Stagnation flow heat transfer

Figure 6 shows the effect of jet velocity on the local heat transfer coefficient for q'' = 0.25 MW m<sup>-2</sup> and  $T_f = 30$  °C. The Hiemenz and Falkner–Skan analyses have shown that the parameter  $C = du_{\delta}/dx$  controls



FIG. 6. Effect of jet velocity on local heat transfer coefficient when q'' = 0.25 MW m<sup>-2</sup> and  $T_f = 30$ °C.

heat transfer at the stagnation line when thermophysical properties are constant (Table 1). Euler's equation for an impinging, finite width, two-dimensional jet with a uniform free stream velocity was evaluated to determine C [44, 45]

$$C = \frac{\mathrm{d}u_{\delta}}{\mathrm{d}x} = \frac{\pi}{4} \frac{V_{\mathrm{j}}}{w_{\mathrm{j}}}.$$
 (2)

Clearly it is the value of  $V_j/w_j$ , not  $V_j$  alone, that should be considered when interpreting the variation of stagnation heat transfer in Fig. 6. Notice, for example, that  $h(x/w_j = 0)$  is nearly the same for  $V_j$  of 1.8 and 2.1 m s<sup>-1</sup>. This result is not surprising because, although the velocities are different,  $V_j/w_j$  is nearly the same (Table 2). An increase in  $V_j/w_j$  causes a corresponding increase in the heat transfer coefficient.

Figure 7 shows the effect of jet temperature on the local heat transfer coefficient for q'' = 0.25 MW m<sup>-2</sup> and  $V_j = 2.5$  m s<sup>-1</sup>. Since the value of C is constant for all data in this figure (C = 230 s<sup>-1</sup>), the only mechanism for changing the stagnation heat transfer coefficient is the temperature dependence of thermal conductivity, viscosity and Prandtl number. With k, v and Pr evaluated at the film temperature ( $(T_w + T_f)/2$ ), the increase of  $h(x/w_j = 0)$  with increasing temperature is in keeping with the trend predicted by Levy's equation (Table 1).

Stagnation flow Nusselt numbers for the data of this study are correlated by

$$Nu_{\rm i} = 0.28 Re_{\rm i}^{0.58} Pr^{0.4}$$
(3)

which is presented in Fig. 8. Thermophysical properties were evaluated at the film temperature, and the range of  $Re_j$  for the data is  $2 \times 10^4 \le Re_j \le 9 \times 10^4$ . Because the Prandtl number range,  $2.7 \le Pr \le 4.5$ , was not sufficient to conclusively establish the Prandtl number exponent, a value of 0.4 was chosen to be consistent with reported results [17, 47, 48].

An expression for the stagnation line Nusselt number can also be derived by substituting equation (2) into equation (1) and multiplying both sides of the results by  $w_i/x$ 

$$Nu_{\rm i} = 0.505 Re_{\rm i}^{1/2} Pr^{0.376}.$$
 (4)

Table 2. Jet velocity, width and stagnation plane velocity gradient for various flow rates when  $w_n = 10.2 \text{ mm}$  and H = 89.7 mm

$G(1 \text{ s}^{-1})$	$V_{\rm n} ({\rm m}{\rm s}^{-1})$	$V_{\rm j}~({\rm m~s^{-1}})$	w <sub>j</sub> (mm)	$V_{\rm j}/w_{\rm j}$	$C/100 (s^{-1})$
1.26	1.23	1.8	6.9	2.6	2.0
1.74	1.69	2.1	8.0	2.6	2.1
2.21	2.13	2.5	8.6	2.9	2.3
3.15	3.05	3.3	9.3	3.5	2.8
3.79	3.66	3.9	9.6	4.1	3.2
4.42	4.28	4.5	9.7	4.6	3.6



FIG. 7. Effect of jet temperature on local heat transfer coefficient when q'' = 0.25 MW m<sup>-2</sup> and  $T_{\rm f} = 30^{\circ}$ C.

This expression is included in Fig. 8. Nusselt numbers computed from equation (3) are 30–50% greater than the analytical results of equation (4) for a laminar, impinging jet. To understand the discrepancy between the stagnation flow heat transfer predicted from first principles and measured values, the effects of free stream turbulence must be considered.

## Turbulence inherent to the flow system

Free stream turbulence is known to enhance heat transfer through a laminar, stagnation flow boundary layer. Experimental variables that affect the enhancement multiplier,  $Nu_j/Nu_0$ , are the free stream turbulence intensity (*Tu*), the Prandtl number (*Pr*), and the velocity gradient normal to the stagnation plane (*C*). The Nusselt number for a turbulence free jet,  $Nu_0$ , can be approximated by equation (4). Enhancement multipliers for the stagnation flow heat transfer data



FIG. 8. Correlation of single phase, stagnation flow heat transfer data,

of this study, which range from 1.33 to 1.51, have been tabulated against Pr, C and  $Re_n$  (Table 3).

Lacking free stream turbulence intensity data, the authors looked for a dependence of Tu on the nozzle Reynolds number evaluated at the free stream temperature. When  $Re_n$  increases, the turbulence intensity at the nozzle discharge is also expected to increase. When variations in Pr are small, the data of Table 3 show a clear trend of increasing enhancement with increasing  $Re_n$ . This result is consistent with widely confirmed reports that, when free stream turbulence intensity increase, stagnation heat transfer rates also increase [19–29].

Clearly, the nozzle Reynolds number cannot be a substitute for free stream turbulence intensity data when correlating stagnation flow heat transfer data, as the relationship between  $Re_n$  and Tu is system specific. Furthermore,  $Re_n$  is coupled to C and, since both are temperature dependent, to the Prandtl number. The sensitivity of  $Nu_i/Nu_0$  to each parameter can therefore not be determined simply by comparing enhancement coefficient data to  $Re_n$ . The mutual dependence of  $Re_n$  and Pr through temperature is particularly troublesome, since opposite variations occur in response to a change in temperature.

Sutera's analytical study [36] showed greater sensitivity of stagnation line heat transfer to free stream

Table 3. Enhancement of stagnation line heat transfer over the value for purely laminar flow

$q''/10^6$ (W m <sup>-2</sup> )	Pr	C/100 (s <sup>-1</sup> )	$Re_{n}/10^{4}$	Nuj	Nu <sub>j</sub> /Nu <sub>o</sub>
0.25	5.43	2.0	1.55	160	1.33
0.49	5.43	2.0	1.55	161	1.33
0.73	5.43	2.0	1.55	162	1.33
0.25	5.43	2.3	2.69	224	1.41
0.49	5.43	2.3	2.69	229	1.43
0.73	5.43	2.3	2.69	229	1.42
1.00	5.43	2.3	2.69	227	1.40
0.25	5.43	3.6	5.41	335	1.49
0.50	5.43	3.6	5.41	335	1.48
1.00	5.43	3.6	5.41	339	1.48
1.23	5.43	3.6	5.41	347	1.51
0.24	3.53	2.3	3.92	222	1.37
0.48	3.53	2.3	3.92	221	1.36
0.25	4.32	2.3	3.92	224	1.39
0.25	5.43	2.1	2.13	187	1.33
0.25	5.43	2.8	3.85	270	1.43
1.00	5.43	2.8	3.85	278	1.43
1.00	5.43	3.2	4.62	308	1.45

turbulence when Pr = 7.0 than when Pr = 0.7. The data of Table 2 are in agreement with this prediction. When the Prandtl number decreases from 5.43 to 3.53, the enhancement coefficient decreases, even though the Reynolds number increases from  $2.7 \times 10^4$  to  $3.9 \times 10^4$ .

Sutera assumed a temperature independent Prandtl number in his analysis. The proper temperature for the Prandtl number data of Table 3 is not obvious. Sensitivity of the enhancement coefficient to Prandtl number can be explained by variations in the relative thermal and hydrodynamic boundary layer thicknesses. This trend suggests the use of a film temperature to characterize Pr. The expected dependence of  $Nu_i/Nu_0$  on Pr, however, was seen only when the Prandtl number was evaluated at the free stream temperature. Over the temperature range considered in this study, Prandtl number values indicate a velocity boundary layer thickness that is greater than the thermal boundary layer thickness (1.5  $\leq \delta/\delta_t \leq$  1.8). Furthermore, enhancement of stagnation flow heat transfer has been attributed to a flow instability that arises close to, but outside of, the laminar, velocity boundary layer [40–43]. When Pr > 1, it is the free stream temperature that characterizes the properties of fluid subject to the instability, which may explain why the Prandtl number based on free stream temperature better predicted variations of  $Nu_i/Nu_0$  than did the Prandtl number based on film temperature.

## Induced turbulence

By introducing a flow disturbance near the nozzle inlet for one set of experiments, the sensitivity of heat transfer to free stream turbulence was investigated. Other measurements were made in which turbulence in the jet was enhanced close to the nozzle discharge.

To determine the effect of perturbing the flow at the nozzle entrance, the honeycomb flow straightener was replaced with a square mesh screen ( $787 \times 787$  wires per meter). With a wire diameter of 0.41 mm and a 0.86 mm square opening, the free flow area of the screen was 46.2% of the total cross-sectional area. Comparative results are shown in Fig. 9. With jet velocities of 1.8 m s<sup>-1</sup> ( $V_n = 1.2$  m s<sup>-1</sup>) and 2.5 m s<sup>-1</sup>



FIG. 9. Effect of perturbing the flow by placing a screen at the nozzle inlet  $(q'' = 0.50 \text{ MW m}^{-2} \text{ and } T_f = 30^{\circ}\text{C})$ .

 $(V_n = 2.1 \text{ m s}^{-1})$ , stagnation heat transfer was only slightly greater when the flow straightener was replaced by the screen. This result is not surprising, since the nozzle was designed to minimize turbulence at the inlet and to partially relaminarize the flow between the inlet and the discharge [44, 45]. Since the velocity at the inlet is only  $V_{\rm n}/5$ , the Reynolds number for a screen wire in cross flow,  $V_{\rm n}d_{\rm w}/v$ , is 120 and 210 when  $V_i$  is 1.8 and 2.5 m s<sup>-1</sup>, respectively. Since turbulence downstream of the screen is reduced as flow is accelerated through the convergent nozzle [49-51], the small increase in stagnation heat transfer resulting from use of the screen is attributed to a small net increase in free stream turbulence intensity at the nozzle discharge when the jet velocity is low. The increase is larger for a screen Reynolds number of 440 ( $V_i = 4.5 \text{ m s}^{-1}$ ,  $V_n = 4.28 \text{ m s}^{-1}$ ), indicating increased turbulence downstream of the screen. The stagnation heat transfer coefficient for this condition is increased 14%.

The effect of perturbing the flow near the nozzle discharge was also investigated. Although screens of various mesh sizes were considered, all of them caused the flow to break into a coarse spray when placed immediately downstream of the nozzle discharge. The jet remained coherent, however, when passing through an array of parallel wires. Experiments in which an array of wires was placed between the nozzle discharge and the heater surface were scaled from a similar experiment described by VanFossen and Simoneau [43], who performed heat transfer and flow visualization studies in the stagnation region of a cylinder in cross flow. The air flow for these experiments was conditioned to maintain the free stream turbulence less than 0.5%. Vorticity was then added to the flow by passing the air over an array of 0.5 mm diameter parallel wires, which had a dimensionless pitch of 12.5 and were placed 547.5 wire diameters upstream of the cylinder. The wires were oriented in cross flow to the free stream but normal to the cylinder's axis, so that the vorticity of the laminar wake behind a wire would have the preferred orientation for amplification. Wakes behind the wires were laminar when the Reynolds number based on wire diameter was less than 120. In this case the authors observed pairs of counter rotating, longitudinal vortices in the stagnation region on the cylinder. When the Reynolds number exceeded 120, the wakes became unstable and longitudinal vortices were not observed.

In this study an array of 0.076 mm diameter wires, with a dimensionless pitch of 10.4 and positioned 500 diameters upstream of the heater, was deployed. Since the wires stretched from prolonged exposure to the jet, experiments were limited to  $V_j = 1.8 \text{ m s}^{-1}$ . The wire Reynolds number was 171, which would indicate a periodic laminar wake if the turbulence intensity upstream of the wire were small. One set of experiments was performed with the wires oriented normal to the impinging jet and parallel to the flow on the heater surface. If laminar wakes form downstream of



FIG. 10. Effect on stagnation heat transfer of superimposing vorticity  $\omega_p$  on  $\omega_j$  when  $V_j = 1.8$  m s<sup>-1</sup> and  $T_f = 30$  °C.

the wires, the vorticity  $\omega_p$  would be oriented in the preferred direction for amplification by stretching of vortex lines. As shown in Fig. 10, the stagnation heat transfer coefficient was approximately 27% greater for this condition than for conditions corresponding to the vorticity  $\omega_j$  associated with the natural free stream turbulence.

Additional measurements were made in which wires were oriented normal to the impinging jet and normal to the flow on the heater surface. If laminar wakes form downstream of the wires, vorticity in the downstream flow,  $\omega_{np}$ , would not be amplified, since the vorticity amplification model indicates increased heat transfer only when vorticity is added in the preferred direction for amplification. In this study, however, differences between stagnation heat transfer for the  $\omega_p$  and  $\omega_{np}$  conditions were small (Fig. 11), suggesting that turbulent wakes behind the wires produce some vorticity in the preferred orientation, regardless of the wire orientation.

Models that relate vortex line stretching to heat transfer enhancement in a stagnation flow indicate a strengthening or weakening of the effect in accordance with an increase or decrease of the velocity gradient. The difference between the local convection coefficient for one level of free stream turbulence and another is, therefore, expected to diminish as  $x/w_j$  increases and the boundary layer remains laminar. This trend is



FIG. 11. Stagnation heat transfer for free stream vorticities of  $\omega_p$  and  $\omega_{np}$  when  $V_j = 1.8 \text{ m s}^{-1}$  and  $T_f = 30^{\circ}\text{C}$ .

clearly seen in Fig. 10 for values of  $x/w_i < 2$ . Downstream results for the  $\omega_p$  and  $\omega_{np}$  flows agree for heat fluxes of 1.00 and 1.23 MW  $m^{-2}$  because vigorous, boiling induced mixing in the boundary layer became the dominant heat transfer mechanism, thereby minimizing the importance of upstream conditions. Since the literature deals only with stagnation flows, differences between convection coefficients in  $\omega_p$  and  $\omega_{np}$ flows for large  $x/w_i$  and low heat flux cannot be explained in terms of the current understanding of heat transfer enhancement by vorticity amplification. Undoubtedly, a three-dimensional laminar boundary layer that is established in the stagnation region will continue to affect heat transfer several jet widths downstream, although specific quantitative and qualitative features of such effects are unknown.

## Heat transfer in the laminar boundary layer

Three flow regimes can be identified in the vicinity of an impinging jet : (i) stagnation flow,  $u_{\delta} = Cx$ , (ii) impingement flow,  $(u_{\delta}/V_j) = f(x/w_j)$ , and (iii) uniform parallel flow,  $(u_{\delta}/V_j) \approx 1$ . An approximate analytical solution to the energy equation for stagnation flow is  $Nu_j = 0.505 Re_j^{1/2} Pr^{0.376}$ , which suggests that  $w_j$  is an appropriate length scale in that regime. However, the streamwise distance measured from a real or effective leading edge is widely accepted as the length scale for uniform parallel flow over a flat plate. Hence, correlation of heat transfer data that spans all three regimes is impeded by the apparent existence of different length scales in the stagnation and parallel flow regimes.

Since proper scales for heat transfer and fluid flow in an impinging jet are difficult to identify, it is not surprising that there are inconsistencies among existing heat transfer correlations for the laminar boundary layer portion of a stagnation flow. For example, McMurray et al. [16] suggests a correlation of the form  $Nu_x = f(Re_{x^*}, Pr)$ , while Zumbrunnen [8] suggests the form  $Nu_i = g(Re_i, Pr, x/w_i)$ . McMurray *et al.* base Nusselt and Reynolds numbers on the distance x, which is measured in the streamwise direction from the stagnation line, and the velocity  $u_{\delta}$ , which is the local free stream velocity just outside of the boundary layer. Although McMurray et al. define a Reynolds number in terms of  $V_i$ , it appears in the correlation only with the multiplier  $\eta = u_{\delta}/V_{i}$ , making  $u_{\delta}$  the effective velocity scale. Zumbrunnen's length scale for *Re* and *Nu*, however, is  $w_i$  and his velocity scale is  $V_i$ .

A scaling analysis, following the methodology of Krantz [52], was performed on the equations that govern fluid flow and heat transfer in the wall jet. The results show that, when  $Re_{x^*}$  and  $Re_{x^*}$   $Pr \ge 100$ , the governing equations reduce to the boundary layer form. Furthermore, the analysis shows that, when the boundary layer equations apply, x and  $u_{\delta}$  are appropriate scales for both the impingement and parallel flow regions.

As shown in Fig. 12, data from this study were correlated by



FIG. 12. Correlation of single phase data in the laminar boundary layer  $(0.25 \le q'' \le 1.00 \text{ MW} \text{ m}^{-2}, 30 \le T_f \le 50^{\circ}\text{C}, 1.8 \le V_j \le 4.5 \text{ m s}^{-1}).$ 

$$Nu_{x} = 0.89 Re_{xx}^{0.48} Pr^{0.4}$$
 (5)

with thermophysical properties evaluated at the film temperature. The range of  $Re_{x^*}$  on which the correlation is based is  $100 \leq Re_{x^*} \leq 10^5$ . Equation (3) should be used to compute the stagnation heat transfer ( $Re_{x^*} < 100$ ), while equation (5) may be used for Reynolds numbers up to the critical value for each experiment. The McMurray *et al.* correlation [16]

$$Nu_x = 0.75 Re_{x^*}^{0.5} Pr^{0.33}$$
(6)

which is also valid in the impingement and parallel flow regions, is in reasonable agreement with equation (5). Since the value of  $Pr^{0.4}/Pr^{0.33}$  was nearly constant for the present study, the McMurray *et al.* correlation could be plotted with equation (5). With increasing heat flux, boiling downstream of the single phase heat transfer region disrupts the boundary layer, thereby reducing  $Re_{x^*c}$  below the value of  $3.6 \times 10^5$  observed for non-boiling flow over the entire heater. The value of  $Re_{x^*c}$  then depends on the position of boiling incipience on the heated surface. To illustrate this phenomenon, some data for  $Re_{x^*} > Re_{x^*c}$ , which clearly deviate from the correlation, were plotted.

The successful use of both  $Nu_x = f(Re_{x^*}, Pr)$  and  $Nu_j = g(Re_j, Pr, x/w_j)$  as correlating functions, in spite of the different length and velocity scales, is readily explained. Since  $u_{\delta}/V_j$  is a function of only  $x/w_j$ , the quantity  $u_{\delta}x/V_jw_j$  can be expressed as some function of  $x/w_j$ . Hence  $Re_{x^*}$  can be written as

$$Re_{x^*} = f\left(\frac{x}{w_j}\right)Re_j \tag{7}$$

and the correlation of laminar flow data can be recast as

$$Nu_{\rm j} = 0.89 \left[ Re_{\rm j} f\left(\frac{x}{w_{\rm j}}\right) \right]^{0.48} Pr^{0.4}.$$
 (8)

Application of the laminar flow correlation, equation (5), requires knowledge of the local free stream velocity,  $u_{\delta}$ , which may be approximated by solving Euler's equation for an impinging, planar, slug flow.

The following function that approximates the solution was derived by the authors [44, 45]

$$\frac{u_{\delta}}{V_{j}} = \frac{\pi}{4} \frac{x}{w_{j}} \quad (0 \le x/w_{j} \le 1)$$
(9a)

$$\frac{u_{\delta}}{V_{j}} = \tanh\left(\frac{x}{w_{j}}\right) \quad (x/w_{j} > 1). \tag{9b}$$

The error incurred by using these expressions approaches zero when  $x/w_j \gg 1$  or  $x/w_j \approx 0$ . The error for  $x/w_j = 1$  is approximately 5% and quickly diminishes to less than 2% for  $x/w_j$  greater or less than 1.

#### SUMMARY

Local convection coefficients were measured for heat transfer from a heated plate to a planar, impinging water jet. One correlation is introduced for the stagnation Nusselt number  $(Nu_j = 0.28 Re_j^{0.58} \times Pr^{0.4})$  and another for the local Nusselt number when  $Re_{x^*} \leq Re_{x^*,c}$   $(Nu_x = 0.89 Re_{x^*}^{0.48} Pr^{0.4})$ . Stagnation convection coefficients were correlated by a least squares fit of the data to the same functional form as the exact, laminar flow solution. Because  $u_{\delta}/V_j$  is a function of  $x/w_j$  only, either  $(x, u_{\delta})$  or  $(w_j, V_r)$  may be used as the length and velocity scales.

Measured stagnation convection coefficients exceeded values predicted for a laminar free stream by factors of 1.3–1.5. Similar results have been widely reported for stagnation flow heat transfer when the free stream is turbulent. Previous experiments show that the enhancement factor  $(Nu_i/Nu_0)$  increases as the free stream turbulence intensity (Tu) increases. Theoretical studies of this phenomenon suggest that the stretching of vortex lines in the impingement region induces three-dimensional flow disturbances just outside of the velocity boundary layer. These vorticity amplification models indicate that the Prandtl number Pr and velocity gradient C in the stagnation region affect heat transfer in addition to the free stream turbulence intensity. Although the free stream turbulence intensity was not measured, qualitative analysis of the data corroborates the dependence of  $Nu_1/Nu_0$  on Tu, Pr and C.

The enhancement of stagnation region cooling by vorticity amplification is system specific. If all parameters except Tu are held constant, cooling will vary significantly from one system to another in response to variations in Tu. New correlating functions are needed that account for vorticity amplification effects. Development of these functions requires that turbulence intensity measurements also be made. Correlations that treat Tu as a variable have been developed for impinging gas jets. The gas jet correlations do not, however, account for the influence of Pr and C on enhancement.

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## TRANSFERT THERMIQUE LOCAL PAR CONVECTION SUR UNE SURFACE POUR UN JET PLAN D'EAU IMPACTANT

**Résumé**—Les distributions de température pariétale et de flux thermique sont mesurées sur une surface plane, tournée vers le haut et à flux constant, refroidie par un jet d'eau. Des résultats sont présentés pour des vitesses entre 1,8 et 4,5 m s<sup>-1</sup>, des températures de fluide de 30, 40 et 50°C et des densités de flux entre 0,25 et 1,00 MW m<sup>-2</sup>. Des échelles appropriées de longueur et de vitesse sont identifiées et les résultats pour les écoulements d'arrêt et de couche limite sont unifiés par des expressions telles que  $Nu = C Re^n Pr^m$ . Des coefficients mesurés de convection au point d'arrêt depassent ceux prédits par une analyse d'écoulement laminaire et les différences sont attribuées à l'existence de la turbulence de l'écoulement libre. Des données sont sensibles aux variations du gradient de vitesse sur la ligne d'arrêt et du nombre de Prandtl qui sont connues affecter l'accroissement du transfert à l'arrêt quand il y a une turbulence d'écoulement libre.

## DER ÖRTLICHE KONVEKTIVE WÄRMEÜBERGANG VON EINER BEHEIZTEN OBERFLÄCHE AN EINEN AUFTREFFENDEN EBENEN WASSERSTRAHL

**Zusammenfassung**—An einer ebenen, gleichmäßig von unten beheizten Oberfläche, die durch einen ebenen auftreffenden Wasserstrahl gekühlt wird, werden die Verteilungen von Oberflächentemperatur und Wärmestromdichte gemessen. Es werden Ergebnisse für Strahlgeschwindigkeiten zwischen 1,8 und 4,5 m s<sup>-1</sup>, Fluidtemperaturen von 30; 40 und 50°C und Wärmestromdichten zwischen 0,25 und 1,0 MW m<sup>-2</sup> vorgestellt. Geeignete Bezugslängen und -geschwindigkeiten werden festgelegt, dies ermöglicht eine Korrelation des Wärmeübergangs in der Staupunkts- und Grenzschichtströmung in folgender Form:  $Nu = C Re^n Pr^m$ . Die gemessenen Wärmeübergangskoeffizienten im Staupunkt überschreiten diejenigen, welche für laminare Strömung berechnet werden—die Ursache wird im Vorhandensein von Freistrahlturbulenz gesehen. Die Ergebnisse hängen stark von Variationen des Geschwindigkeitsgradienten in der Staulinie und der Prandtl-Zahl ab. Dies sind bekannte Einflüsse auf den Wärmeübergang im Staugebiet bei Freistrahlturbulenz.

## ЛОКАЛЬНЫЙ КОНВЕКТИВНЫЙ ТЕПЛОПЕРЕНОС МЕЖДУ НАГРЕТОЙ ПОВЕРХНОСТЬЮ И ПАДАЮЩЕЙ ПЛОСКОЙ ВОДЯНОЙ СТРУЕЙ

Аннотация Выполнены измерения распределений температур и тепловых потоков на плоской обращенной вверх поверхности с постоянным тепловым потоком, охлаждаемой плоской падающей водяной струей. Приведены данные для исменения скорости струи в интервале от 1,8 до 4,5 м с<sup>-1</sup>, температуры жидкости величиной 30, 40, 50°С и изменения теплового потока в интервале от 0,25 до 1,00 MBT м<sup>-2</sup>. Определены соответствующие масштабы длины и скорости, и с помощью выражений вида  $Nu = C Re^n Pr^m$  согласованы результаты для течений в зоне торможения и пограничном слое. Значения экспериментально определенных коэффициентов конвекции в зоне торможения о ламинарности течения. Это различие объясняется существованием турбулентности свободного потока. Установлена зависимость полученных данных от градиента линейной скорости в зоне торможения при воне теплопереноса в зоне торможения при наличи турбулентности свободного потока.